Q No		Solution	Marks	Remarks
1	(a)	$\sum_{r=0}^{n}\frac{f^{r}(a)}{r!}(x-a)^{r}$ is defined as the Taylor polynomial of $f(x)$ about a. $f(x)=\ln(1-x+x^{2})$ $f^{1}(x)=\frac{2x-1}{1-x+x^{2}}$ $(1-x+x^{2})f^{1}(x)=2x-1$ $(1-x+x^{2})f^{2}(x)+(2x-1)f^{1}(x)=2$ $(1-x+x^{2})f^{2}(x)+(2x-1)f^{1}(x)-2=0$ $(1-x+x^{2})f^{3}(x)+(2x-1)f^{2}(x)$ $+(2x-1)f^{2}(x)+2f^{1}(x)=0$ $(1-x+x^{2})f^{3}(x)+2(2x-1)f^{2}(x)+2f^{1}(x)=0$ $(1-x+x^{2})f^{4}(x)+(2x-1)f^{3}(x)+2(2x-1)f^{3}(x)$ $+4f^{2}(x)+2f^{2}(x)=0$ $(1-x+x^{2})f^{4}(x)+3(2x-1)f^{3}(x)+6f^{2}(x)=0$ $(1-x+x^{2})f^{5}(x)+(2x-1)f^{4}(x)+3(2x-1)f^{4}(x)$ $+6f^{3}(x)+6f^{3}(x)=0$ $(1-x+x^{2})f^{5}(x)+4(2x-1)f^{4}(x)+12f^{3}(x)=0$ $f(0)=0$, $f^{1}(0)=-1$, $f^{2}(0)=1$, $f^{3}(0)=4$, $f^{4}(0)=6$, $f^{5}(0)=-24$ Fifth order Taylor polynomial of $f(x)$ .is $\sum_{r=0}^{5}\frac{f^{r}(0)}{r!}x^{r}=0+\frac{(-1)x}{1!}+\frac{(1)x^{2}}{2!}+\frac{(4)x^{3}}{3!}+\frac{(6)x^{4}}{4!}-\frac{(24)x^{5}}{5!}$ $=-x+\frac{1}{2}x^{2}+\frac{2}{3}x^{3}+\frac{1}{4}x^{4}-\frac{1}{5}x^{5}$		
	(b)	$g(x)=\ln(1-x)$, $g(x)=\frac{-1}{1-x}$, $g^{2}(x)=\frac{(-1)(-1)(-1)}{(1-x)^{2}}=-\frac{1}{(1-x)^{2}}$ $g^{3}(x)=\frac{(-1)(-2)(-1)}{(1-x)^{3}}=-\frac{1.2}{(1-x)^{3}}=-\frac{2!}{(1-x)^{3}}$ $g^{4}(x)=-\frac{2!(-3)(-1)}{(1-x)^{4}}=-\frac{3!}{(1-x)^{4}}$ $g^{5}(x)=-\frac{3!(-4)(-1)}{(1-x)^{5}}=-\frac{4!}{(1-x)^{5}}$		

Q No		Solution	Marks	Remarks
		Similarly in the above manner $g^r(x) = -\frac{(r-1)!}{(1-x)^r}$ for $r \ge 1$ and $r \in Z^+$ $\ln(1-x) = \sum_{r=0}^{\propto} \frac{x^r g^r(0)}{r!}$ $\ln(1-x) = \sum_{r=1}^{\propto} \frac{x^r g^r(0)}{r!}$ since $g(0) = 0$ $\ln(1-x) = \sum_{r=0}^{\propto} \frac{x^r(-(r-1)!)}{r!}$ $\ln(1-x) = -\sum_{r=0}^{\propto} \frac{x^r}{r}$ $\ln(1-x) + \sum_{r=0}^{\propto} \frac{x^r}{r} = 0$ $\ln(1+3x-4x^2) = \ln(1+4x)(1-x)$ Since $(1-4x)(1-x) > 0$ $-\frac{1}{4} < x < 1$ $\ln(1+3x-4x^2) = \ln(1+4x) + \ln(1-x)$ $\lvert 4x \rvert < 1 \Leftrightarrow \lvert x \rvert < \frac{1}{4}$ and $\lvert x \rvert < 1 \therefore \lvert x \rvert < \frac{1}{4}$ $\ln(1+3x-4x^2) = -\sum_{r=1}^{\propto} \frac{(-4x)^r}{r} - \sum_{r=1}^{\propto} \frac{x^r}{r}$ $= \sum_{r=1}^{\propto} \frac{(-1)^{r+1} 4^r x^r}{r} - \sum_{r=1}^{\propto} \frac{x^r}{r} = \sum_{r=1}^{n} [(-1)^{r+1} 4^r - 1] \frac{x^r}{r}$ Let $x = \frac{1}{2}$ $\ln\left(1 - \frac{1}{2}\right) + \sum_{r=0}^{\propto} \frac{1}{r} \left(\frac{1}{2}\right)^r = 0$ $\ln 1 - \ln 2 + \sum_{r=0}^{\propto} \frac{1}{r} \left(\frac{1}{2}\right)^r = 0$ $\ln 2 = \sum_{r=0}^{\propto} \frac{1}{r 2^r}$		

QNo		Answer	Marks	Remarks
2	(a)	$f(t) = \begin{cases} 1 & 0 < t < 4 \\ t^2 & 4 \le t \end{cases}$ $L(f(t)) = \int_0^{\propto} e^{-st} f(t) dt$ $L(f(t)) = \int_0^4 e^{-st} 1 dt + \int_4^{\propto} e^{-st} t^2 dt$		

Q No		Solution	Marks	Remarks
		$=\left[\frac{e^{-st}}{-s}\right]_0^4 + \left[\frac{t^2}{-s}e^{-st} - \frac{2t}{(-s)^2}e^{-st} + \frac{2}{(-s)^3}e^{-st}\right]_4^{\infty}$ $=\left[\frac{1}{s}-\frac{e^{-4s}}{s}\right]+\left[\frac{16}{s}e^{-4s}+\frac{8}{s^2}e^{-4s}+\frac{2}{s^3}e^{-4s}\right]$ $=\frac{1}{s}+\left(\frac{15}{s}+\frac{8}{s^2}+\frac{2}{s^3}\right)e^{-4s}$		
	(b	$f * g = \int_0^t f(u)g(t-u)du$ $x(t) = e^t + 2\int_0^t \sin 2(t-u)x(u)du$ Taking the Laplace transformation of both sides $X(s) = \frac{1}{s-1} + \frac{2}{s^2+4}X(s)$ $X(s) = \frac{s^2+4}{(s-1)(s^2+2)}$ $X(s) = \frac{5}{3}\left(\frac{1}{s-1}\right) - \frac{2}{3}\left(\frac{s}{s^2+2}\right) - \frac{2}{3\sqrt{2}}\left(\frac{\sqrt{2}}{s^2+2}\right)$ Taking the inverse Laplace transformation of both sides $x(t) = \frac{5}{3}e^t - \frac{2}{3}\cos\sqrt{2}t - \frac{2}{3\sqrt{2}}\sin\sqrt{2}t$		
QNo		Answer	Marks	Remarks
3	(a)	$\frac{d^2y}{dx^2} + y = 2\sin 2t$, where $t \geq 0$ and $y(0)=1,\ y'(0)=2$ $L\left(\frac{d^2y}{dx^2}\right) = s^2Y(s) - sy(0) - y'(0)$ Taking the Laplace transformation of both sides $s^2Y - s.1 - 2 + Y = 2\frac{2}{s^2+4}$ $Y = \frac{4}{(s^2+1)(s^2+4)} + \frac{s+2}{s^2+1}$ $\frac{4}{(s^2+1)(s^2+4)} = \frac{A}{s^2+1} + \frac{B}{s^2+4}$ $4 = A(s^2+4) + B(s^2+1)$ $A = \frac{4}{3}$ and $B = -\frac{4}{3}$		

Q No		Solution	Marks	Remarks
		$Y=\frac{4}{3(s^2+1)}-\frac{4}{3(s^2+4)}+\frac{s}{s^2+1}+\frac{2}{s^2+1}$ $Y=\frac{10}{3(s^2+1)}-\frac{4}{3(s^2+4)}+\frac{s}{s^2+1}$ Taking the inverse Laplace transformation of both sides $y=\frac{10}{3}\sin t-\frac{2}{3}\sin 2t+\cos t$		
	(b)	$\frac{dx}{dt}-6x+3y=8e^t \quad \frac{dy}{dt}-2x-y=4e^t$, where $x(0)=1$ and $y(0)=0$. $L\left(\frac{dy}{dx}\right)=sF(s)-f(0)$ Taking the Laplace transformation of both sides $sX(s)-x(0)-6X(s)+3Y(s)=8\frac{1}{s-1}$ $(s-6)X-1+3Y=\frac{8}{s-1}$ $(s-6)X+3Y=\frac{s+7}{s-1}$……[1] $sY(s)-y(0)-2X(s)-Y(s)=4\frac{1}{s-1}$ $(s-1)Y-2X=\frac{4}{s-1}$………[2] From [2] $X=\frac{(s-1)}{2}Y-\frac{4}{2(s-1)}$ Substituting in [1] $(s-6)\left[\frac{(s-1)}{2}Y-\frac{4}{2(s-1)}\right]+3Y=\frac{s+7}{s-1}$ $Y=\frac{2(3s-5)}{(s-1)(s-3)(s-4)}$ $Y=\frac{-2}{3(s-1)}-2\frac{2}{s-3}+\frac{14}{3(s-4)}$ Taking the inverse Laplace transformation of both sides $y=-\frac{2}{3}e^t-4e^{3t}+\frac{14}{3}e^{4t}$ $x=\frac{-4}{2(s-1)}+\frac{(s-1)}{2}\frac{2(3s-5)}{(s-1)(s-3)(s-4)}$		

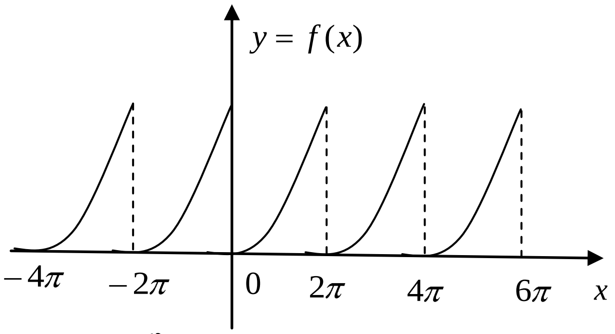
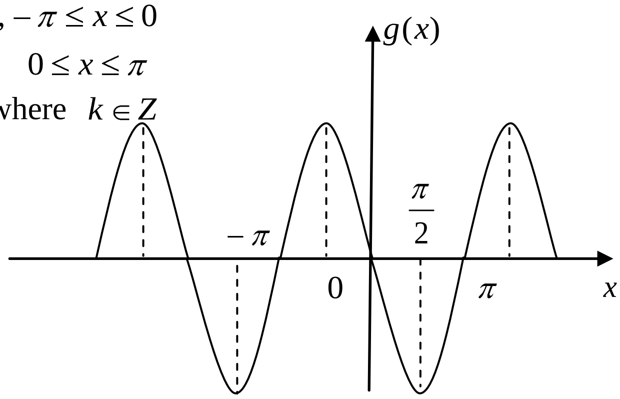
Q No		Solution	Marks	Remarks
		$X = \frac{-4}{2(s-1)} + \frac{1}{2}\frac{2(3s-5)}{(s-3)(s-4)}$ $\frac{(3s-5)}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$ $3s-5 = A(s-4) + B(s-3)$ $A = -4$ and $B = 7$ $X = \frac{-4}{2(s-1)} + \left(\frac{-4}{(s-3)} + \frac{7}{(s-4)}\right)$ $X = \frac{-4}{2(s-1)} - \frac{4}{(s-3)} + \frac{7}{(s-4)}$ Taking the inverse Laplace transformation of both sides $x = -2e^{t} - 4e^{3t} + 7e^{4t}$		

Q No		Solution	Marks	Remarks
4		$\frac{1}{D-3\alpha} f(x) = e^{3x} \frac{1}{D} e^{3x} f(x)$ Let $y = \frac{1}{D-3\alpha} f(x)$ Operating both sides by $(D-3\alpha)$ $(D-3\alpha)y = f(x)$ $\frac{dy}{dx} - 3\alpha y = f(x)$ The integrating factor $e^{\int -3\alpha dx} = e^{-3\alpha x}$ $e^{-3\alpha x}\frac{dy}{dx} - 3\alpha e^{-3\alpha x} y = e^{-3\alpha x} f(x)$ $\frac{d}{dx}(e^{-3\alpha x} y) = e^{-3\alpha x} f(x)$ $ye^{-3\alpha x} = \int e^{-3\alpha x} f(x) dx$ $y = e^{3\alpha x}\int e^{-3\alpha x} f(x) dx \; \frac{1}{D-3\alpha} f(x) = e^{3x} \frac{1}{D} e^{3x} f(x)$		
		$(D^2 - 12D + 32)y = 12\sin 3x$ Particular integral $y_p = \frac{1}{D^2 - 12D - 32} 12\sin 3x$ $y_p = \frac{1}{(D-8)(D-4)} 12\sin 3x$ $\frac{1}{(D-8)(D-4)} = \frac{A}{(D-8)} + \frac{B}{(D-4)}$		

Q No		Solution	Marks	Remarks
		$1 = A(D-4) + B(D-8)\quad A = \frac{1}{4}$ and $B = -\frac{1}{4}$ $y_p = \frac{1}{4}\left(\frac{1}{(D-8)} - \frac{1}{(D-4)}\right)12\sin 3x = \frac{12}{4}\left(\frac{1}{D-8}\sin 3x - \frac{1}{D-4}\sin 3x\right)$ $= 3\left(e^{8x}\frac{1}{D}e^{-8x}\sin 3x - e^{4x}\frac{1}{D}e^{-4x}\sin 3x\right)$ $= 3\left(e^{8x}\int e^{-8x}\sin 3x dx - e^{4x}\int e^{-4x}\sin 3x dx\right)$ $= 3e^{8x}\frac{e^{-8x}}{3^2+(-8)^2}[-8\sin 3x - 3\cos 3x] - 3e^{4x}\frac{e^{-4x}}{3^2+(-4)^2}[-4\sin 3x - 3\cos 3x]$ $y_p = \frac{276}{1825}\sin 3x + \frac{432}{1825}\cos 3x$ The characteristics equation is $\lambda^2 - 12\lambda + 32 = 0$ $(\lambda-8)(\lambda-4) = 0\quad \lambda = 8$ or $\lambda = 4$ The complimentary function $y_c = Ae^{8x} + Be^{4x}$ $\therefore$ The general solution $y = Ae^{8x} + Be^{4x} + \frac{276}{1825}\sin 3x + \frac{432}{1825}\cos 3x$, where A and B are arbitrary constants.		

QNo		Solution	Marks	Remarks
5	(a)	$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 6xe^{-x}$ Let $y_T = (Ax+B)e^{-x}$, where A and B are constants. $\frac{dy_T}{dx} = -(Ax+B)e^{-x} + Ae^{-x}\quad \frac{dy_T}{dx} = -(Ax+B-A)e^{-x}$ $\frac{d^2y_T}{dx^2} = (Ax+B-A)e^{-x} - Ae^{-x}\quad \frac{d^2y_T}{dx^2} = (Ax+B-2A)e^{-x}$ Substituting in the differential equation $(Ax+B-2A)e^{-x} + 3(Ax+B-A)e^{-x} + 2(Ax+B)e^{-x} = 6xe^{-x}$ Since $e^{-x} \neq 0\quad 6Ax - 5A + 6B = 6x$ $A = 1$ and $B = \frac{5}{6}\quad y_T = \left(x + \frac{5}{6}\right)e^{-x}$ The characteristic Equation $\lambda^2 - 3\lambda + 2 = 0\quad \lambda = 2$ or $\lambda = 1$ The complimentary function $y = \alpha e^{x} + \beta e^{2x}$ The general solution $y = \alpha e^{x} + \beta e^{2x} + \left(x + \frac{5}{6}\right)e^{-x}$, where α and β are arbitrary constants		

Q No		Solution	Marks	Remarks
	(b)	$\frac{dx}{dt} = 5x - 3y \quad \frac{dy}{dt} = x + 2y$ Let $x = \alpha e^{\mu t}$ and $y = \beta e^{\mu t}$ $\frac{dx}{dt} = \alpha \mu e^{\mu t}$ and $\frac{dy}{dt} = \beta \mu e^{\mu t}$ Substituting in the differential equations we get $\alpha \mu e^{\mu t} = 5\alpha e^{\mu t} - 3\beta e^{\mu t}$ Since $e^{\mu t} \neq 0 \quad (\mu - 5)\alpha + 3\beta = 0$.........[1] $\beta \mu e^{\mu t} = \alpha e^{\mu t} + 2\beta e^{\mu t}$ Since $e^{\mu t} \neq 0 \quad -\alpha + (\mu - 2)\beta = 0$.........[2] For non- trivial solutions of α and β $\begin{vmatrix} \mu - 5 & 3 \\ -1 & \mu - 2 \end{vmatrix} = 0 \quad \mu = \frac{7}{2} + \frac{\sqrt{3}}{2} i$ or $\mu = \frac{7}{2} - \frac{\sqrt{3}}{2} i$ When $\mu = \frac{7}{2} + \frac{\sqrt{3}}{2} i \quad \left(\frac{7}{2} + \frac{\sqrt{3}}{2} i - 5\right)\alpha_1 + 3\beta_1 = 0 \; \therefore \; \beta_1 = -\frac{\sqrt{3}}{6}(\sqrt{3} - i)\alpha_1$ When $\mu = \frac{7}{2} - \frac{\sqrt{3}}{2} i \quad \left(\frac{7}{2} - \frac{\sqrt{3}}{2} i - 5\right)\alpha_2 + 3\beta_2 = 0 \; \therefore \; \beta_2 = \frac{\sqrt{3}}{6}(\sqrt{3} + i)\alpha_2$ $x = \alpha_1 e^{\left(\frac{7}{2} + \frac{\sqrt{3}}{2} i\right)t} + \alpha_2 e^{\left(\frac{7}{2} - \frac{\sqrt{3}}{2} i\right)t}$ $y = \frac{\sqrt{3}}{6}(\sqrt{3} - i)\alpha_1 e^{\left(\frac{7}{2} + \frac{\sqrt{3}}{2} i\right)t} + \frac{\sqrt{3}}{6}(\sqrt{3} - i)\alpha_2 e^{\left(\frac{7}{2} - \frac{\sqrt{3}}{2} i\right)t}$ $\left(\frac{7}{2} + \frac{\sqrt{3}}{2} i - 5\right)\alpha_1 + 3\beta_1 = 0 \; \therefore \; \beta_1 = -\frac{\sqrt{3}}{6}(\sqrt{3} - i)\alpha_1$ When $\mu = \frac{7}{2} - \frac{\sqrt{3}}{2} i$ $\left(\frac{7}{2} - \frac{\sqrt{3}}{2} i - 5\right)\alpha_2 + 3\beta_2 = 0 \; \therefore \; \beta_2 = \frac{\sqrt{3}}{6}(\sqrt{3} + i)\alpha_2$ $x = \alpha_1 e^{\left(\frac{7}{2} + \frac{\sqrt{3}}{2} i\right)t} + \alpha_2 e^{\left(\frac{7}{2} - \frac{\sqrt{3}}{2} i\right)t}$ $y = \frac{\sqrt{3}}{6}(\sqrt{3} - i)\alpha_1 e^{\left(\frac{7}{2} + \frac{\sqrt{3}}{2} i\right)t} + \frac{\sqrt{3}}{6}(\sqrt{3} - i)\alpha_2 e^{\left(\frac{7}{2} - \frac{\sqrt{3}}{2} i\right)t}$		

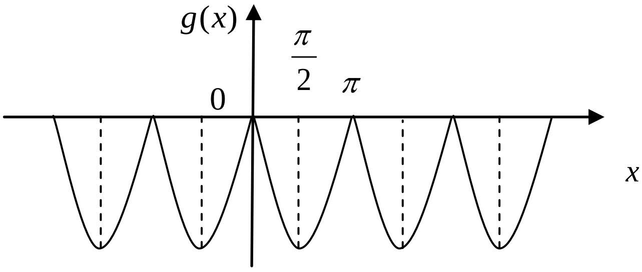
Q.No		Solutions	Marks	Remarkes
6	(a)	$f(x)=x^2$, $0\le x\le 2\pi$ and $f(x+2k\pi)=f(x)$ where $k\in Z$		
				
		$f(x)=\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$		
		$a_0=\dfrac{1}{\pi}\int\limits_0^{2\pi}f(x)dx=\dfrac{1}{\pi}\int\limits_0^{2\pi}x^2dx=\dfrac{1}{\pi}\left[\dfrac{x^3}{3}\right]_0^{2\pi}=\dfrac{8\pi^2}{3}$		
		$a_n=\dfrac{1}{\pi}\int\limits_0^{2\pi}f(x)\cos nx dx=\dfrac{1}{\pi}\int\limits_0^{2\pi}x^2\cos nx dx$		
		$=\dfrac{1}{\pi}\left[\dfrac{x^2}{n}\sin nx+\dfrac{2x}{n^2}\cos nx-\dfrac{2}{n^3}\sin nx\right]_0^{2\pi}=\dfrac{4}{n^2}$		
		$b_n=\dfrac{1}{\pi}\int\limits_0^{2\pi}f(x)\sin nx dx=\dfrac{1}{\pi}\int\limits_0^{2\pi}x^2\sin nx dx$		
		$=\dfrac{1}{\pi}\left[-\dfrac{x^2}{n}\cos nx+\dfrac{2x}{n^2}\sin nx-\dfrac{2}{n^3}\cos nx\right]_0^{2\pi}=-\dfrac{4\pi}{n}$		
		$x^2=\dfrac{4\pi^2}{3}+\sum\limits_{n=1}^{\infty}\left(\dfrac{4}{n^2}x\cos nx-\dfrac{4\pi}{n}\sin nx\right)$		
		$x^2=\dfrac{4\pi^2}{3}+4\sum\limits_{n=1}^{\infty}\left(\dfrac{1}{n^2}x\cos nx-\dfrac{\pi}{n}\sin nx\right)$		
		At $x=0$ $\dfrac{1}{2}\left[\lim\limits_{x\to0^+}f(x)+\lim\limits_{x\to0^-}f(x)\right]=2\pi^2$		
		$\dfrac{4\pi^2}{3}+4\sum\limits_{n=1}^{\infty}\left(\dfrac{1}{n^2}\right)=2\pi^2$ $\quad\therefore\sum\limits_{n=1}^{\infty}\left(\dfrac{1}{n^2}\right)=\dfrac{\pi^2}{6}$		
	(b)	$g(x)=x(x-\pi)$, $0\le x\le\pi$ as a odd function		
		$g(x)=\begin{cases}-x(x+\pi)\,, & -\pi\le x\le 0\\ x(x-\pi)\,, & 0\le x\le\pi\end{cases}$		
		$g(x+2k\pi)=g(x)$ where $k\in Z$		
				

Q.No.	Solution	Marks	Remakes

$g(x)=x(x-\pi)$, $0\le x\le \pi$ as a even function

$$g(x)=\begin{cases} x(x+\pi)\,, -\pi\le x\le 0 \\ x(x-\pi)\,, \quad 0\le x\le \pi \end{cases}$$

$g(x+2k\pi)=g(x)$ where $k\in Z$



If $g(x)$ odd $g(x)=\sum_{n=1}^{\infty} b_n \sin nx$

$$b_n=\frac{2}{\pi}\int_0^{\pi} g(x)\sin nx dx$$

$$b_n=\frac{2}{\pi}\int_0^{\pi} x(x-\pi)\sin nx dx$$

$$=\frac{2}{\pi}\left[-\frac{(x^2-\pi x)}{n}\cos nx+\frac{(2x-\pi)}{n^2}\sin nx+\frac{2}{n^3}\cos nx\right]_0^{\pi}$$

$$=\frac{2}{\pi}\left[\frac{2}{n^3}((-1)^n-1)\right]=\frac{4}{\pi n^3}((-1)^n-1)$$

$$x(x-\pi)=\sum_{n=1}^{\infty}\frac{4}{\pi n^3}((-1)^n-1)\sin nx$$

$$=\frac{4}{\pi}\left[-\frac{2}{1^3}\sin x-\frac{2}{3^3}\sin 3x-\frac{2}{5^3}\sin 5x.........\right]$$

$$\therefore \quad x(x-\pi)=-\frac{8}{\pi}\sum_{n-1}^{\infty}\frac{\sin(2n-1)x}{(2n-1)^3}$$

If $g(x)$ even $g(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx$

$$a_0=\frac{2}{\pi}\int_0^{\pi} g(x)dx=\frac{2}{\pi}\int_0^{\pi} x(x-\pi)dx=-\frac{\pi^3}{3}$$

$$a_n=\frac{2}{\pi}\int_0^{\pi} g(x)\cos nx dx$$

$$a_n=\frac{2}{\pi}\int_0^{\pi} x(x-\pi)\cos nx dx$$

$$=\frac{2}{\pi}\left[\frac{(x^2-\pi x)}{n}\sin nx+\frac{(2x-\pi)}{n^2}\cos nx-\frac{2}{n^3}\sin nx\right]_0^{\pi}$$

$$=\frac{2}{\pi}\left[\frac{\pi}{n^2}((-1)^n+1)\right]=\frac{2}{n^2}((-1)^n+1)$$

Q. No		Solution	Marks	Remarks
		$x(x-\pi)=-\frac{\pi^3}{6}+\sum_{n=1}^{\infty}\frac{2}{n^2}((-1)^n+1)\cos nx$ $=-\frac{\pi^3}{6}+2\left[\frac{2}{2^2}\cos x+\frac{2}{4^2}\cos 3x+\frac{2}{6^2}\cos 5x.........\right]$ $\therefore \quad x(x-\pi)=-\frac{\pi^3}{6}+\frac{1}{2}\sum_{n-1}^{\infty}\frac{\cos 2nx}{n^2}$		

Q No		Solution	Marks	Remarks
7	(a)	$x+y=2$ $x-y=0$ $3x+y=t$ The augmented matrix of the above system is $\begin{bmatrix}1 & 1 & 2\\ 1 & -1 & 0\\ 3 & 1 & t\end{bmatrix}$ $R_2\to R_2-R_1 \quad \downarrow \quad R_3\to R_3-3R_1$ $\begin{bmatrix}1 & 1 & 2\\ 0 & 2 & 2\\ 0 & -2 & t-6\end{bmatrix}$ $R_2\to\frac{1}{2}R_2 \quad \downarrow \quad R_3\to R_3+R_2$ $\begin{bmatrix}1 & 1 & 2\\ 0 & 1 & 1\\ 0 & 0 & t-4\end{bmatrix}$ Hence $t\neq 4$ the system is inconsistent. That is no solution of the system. If $t=4$ the system is consistent and it has a unique solution When $t=4$ $\begin{bmatrix}1 & 1 & 2\\ 0 & 1 & 1\\ 0 & 0 & 0\end{bmatrix} R_1\to \quad R_1-R_2 \quad \downarrow \quad \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 0 & 0 & 0\end{bmatrix}$ $\therefore$ The solution is $x=1$ and $y=1$		

Q.No.		Solution	Marks	Coments
(b)				

$x-2y+3z=4$

$2x-3y-az=5$

$3x-4y+5z=b$

The augmented matrix of the above system is

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 2 & -3 & a & 5 \\ 3 & -4 & 5 & b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 2 & -3 & a & 5 \\ 3 & -4 & 5 & b \end{bmatrix}$$

$R_2 \to R_2 - 2R_1 \quad \downarrow \quad R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & a-6 & -3 \\ 0 & 2 & -4 & b-12 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2 \quad \downarrow \qquad R_1 \to R_1 + 2R_2 + R_3 \qquad R_2 \to \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & a-6 & -3 \\ 0 & 0 & 8-2a & b-6 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 & b-8 \\ 0 & 1 & -2 & \frac{b-12}{2} \\ 0 & 0 & 8-2a & b-6 \end{bmatrix}$$

If $a \neq 4$

$R_3 \to \frac{1}{-2(a-4)}R_3 \qquad R_1 \to R_1 + R_3 \qquad R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & -1 & b-8 \\ 0 & 1 & -2 & \frac{b-12}{2} \\ 0 & 0 & 1 & \frac{b-6}{2(4-a)} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & b-8+\frac{b-6}{8-2a} \\ 0 & 1 & 0 & \frac{b-12}{2}+\frac{b-6}{4-a} \\ 0 & 0 & 1 & \frac{b-6}{2(4-a)} \end{bmatrix}$$

If $a \neq 4$ the system has a solution

$$x = b-8+\frac{b-6}{8-2a} \qquad y = \frac{b-12}{2}+\frac{b-6}{4-a} \qquad z = \frac{b-6}{8-2a}$$

Where a and b are parameters such that $a \neq 4$

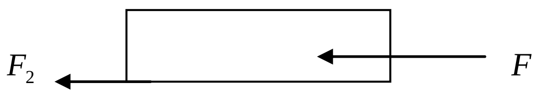
If $a = 4$

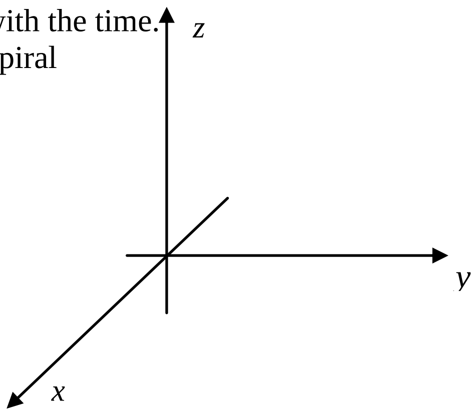
$$\begin{bmatrix} 1 & 0 & -1 & b-8 \\ 0 & 1 & -2 & \frac{b-12}{2} \\ 0 & 0 & 0 & b-6 \end{bmatrix}$$

In this case if $b \neq 6$ the system has no solutions that are the system is inconsistent.

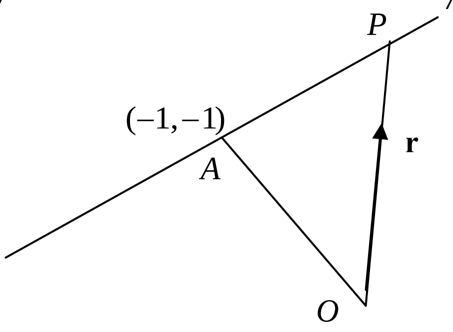
Q.No.	Solution	Marks	Comments
	If $b = 6$ $\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $x - z = -2$ and $y - 2z = -3$ Take $z = \lambda$ where is a parameter Then the solution is $x = \lambda - 2$, $y = 2\lambda - 3$ and $z = \lambda$		

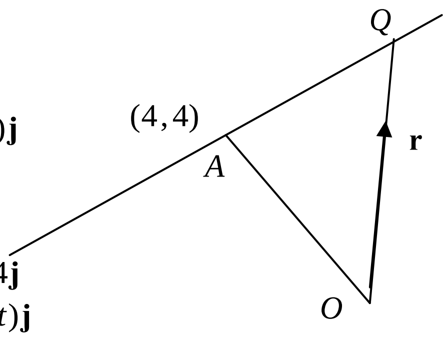
Q No	Solution	Marks	Remarks
8	 $F_2 = Mkv^2$ and F_1 = driven foce Applying $H = FV$ $P = F_1 v$ $F_1 = \frac{P}{v}$ Applyng $\mathbf{F} = m\mathbf{a}$ $F_1 - F_2 = Mv\frac{dv}{dt}$ $\frac{P}{v} - Mkv^2 = Mv\frac{dv}{dx}$ The maximum speed is attained when $v\frac{dv}{dx} = 0$ $\frac{P}{V} - MkV^2 = 0 \quad k = \frac{P}{MV^3}$ $\frac{P}{v} - \frac{P}{V^3}v^2 = Mv\frac{dv}{dx}$ $\frac{P(V^3 - v^3)}{V^3 v} = Mv\frac{dv}{dx}$ $dx = \frac{Mv^2V^3}{P(V^3 - v^3)}dv$ $x = -\frac{MV^3}{3P}\ln(V^3 - v^3) + constant$ When $x = 0 \quad v = 0$ $constant = \frac{MV^3}{3P}\ln V^3$		

Q No		Solution	Marks	Remarks
		$x=-\frac{MV^3}{3P}\ln(V^3-v^3)+\frac{MV^3}{3P}\ln V^3$ $\frac{1}{3}\ln\left(\frac{V^3}{V^3-v^2}\right)=\frac{Px}{MV^3}$ $\frac{V^3}{V^3-v^2}=e^{\frac{3Px}{MV^3}}=-e^{3kx}$ Let $v=\frac{V}{2}$ when $x=x_1$ $\quad x_1=\frac{1}{2k}\ln\left(\frac{8}{7}\right)$  Applying $\mathbf{F}=m\mathbf{a}$ $M\frac{dv}{dt}=-F-Mkv^2 \quad \frac{dv}{\frac{F}{Mk}+v^2}=-dt$ $\int_{\frac{V}{2}}^{0}\frac{dv}{\frac{F}{Mk}+v^2}=-\int_0^t dt=-t \quad \int_0^{\frac{V}{2}}\frac{dv}{\frac{F}{Mk}+v^2}=t$ $t=\left[\frac{1}{\sqrt{\frac{F}{Mk}}}\tan^{-1}\left(\frac{v}{\sqrt{\frac{F}{Mk}}}\right)\right]_0^{\frac{V}{2}}$ $=\sqrt{\frac{Mk}{F}}\tan^{-1}\left(\frac{V}{2}\sqrt{\frac{Mk}{F}}\right)$		

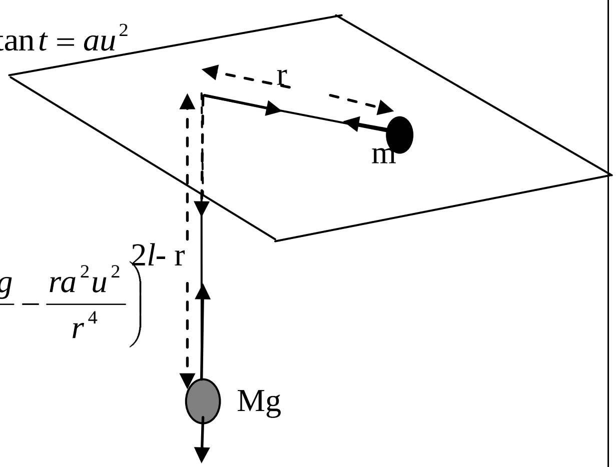
Q No		Solution	Marks	Remarks
		$\mathbf{r}=5\sin 2t\mathbf{i}+12\cos 2t\mathbf{j}+3t\mathbf{k}$ Let $x=5\sin 2t$, $y=12\cos 2t$ and $z=3t$ $\therefore \frac{x^2}{5^2}+\frac{y^2}{12^2}=1$ Always x and y co – ordinates lie on an ellipse while z co – ordinate increases with the time. $\therefore$ the point moves on an elliptic spiral  		

\Q.No		Solution	Marks	Comments
9	(a)	(ii) Velocity $\dot{\mathbf{r}} = 10\cos 2t\mathbf{i} - 24\sin 2t\mathbf{j} + 3\mathbf{k}$ $\left\lvert\dot{\mathbf{r}}\right\rvert = \sqrt{10^2\cos^2 2t + 24^2\sin^2 2t + 3^2} = \sqrt{100 + 476\sin^2 2t + 9}$ $\max\left\lvert\dot{\mathbf{r}}\right\rvert = \sqrt{100 + 476 + 9} = 3\sqrt{65}$ $\min\left\lvert\dot{\mathbf{r}}\right\rvert = \sqrt{100 + 0 + 9} = \sqrt{109}$ Acceleration $\ddot{\mathbf{r}} = -20\sin 2t\mathbf{i} - 48\cos 2t\mathbf{j}$ $\left\lvert\ddot{\mathbf{r}}\right\rvert = \sqrt{20^2\sin^2 2t + 48^2\cos^2 2t} = 4\sqrt{25 + 119\cos^2 2t}$ $= \sqrt{25+119\cos^2 2t}$ $\max\left\lvert\ddot{\mathbf{r}}\right\rvert = 48$ and $\min\left\lvert\ddot{\mathbf{r}}\right\rvert = 20$		

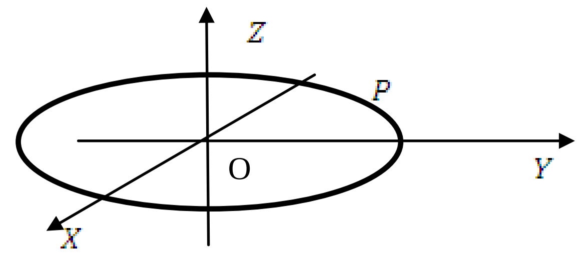
Q No.		Solution	Marks	Comments
9	(b)	$\overrightarrow{OC} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, $\overrightarrow{OD} = -4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ $\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD} = -(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) + (-4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ $\overrightarrow{CD} = -6\mathbf{i} - \mathbf{j} + 6\mathbf{k}$ $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} + 6\mathbf{j}$ $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -(4\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) + 2\mathbf{i} + 6\mathbf{j}$ $\overrightarrow{AB} = -2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ $\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{2^2 + 3^2 + 6^2} = 7$ $\therefore$ The unit vector along the direction $\overrightarrow{AB}$ is $\frac{1}{7}(-2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$ $8\,N$ force along the direction $\overrightarrow{AB}$ is $\frac{8}{7}(-2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$ Woke done by the force is $\mathbf{F}\cdot\mathbf{S}$ $\mathbf{F}\cdot\mathbf{S} = \frac{8}{7}(-2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})\cdot(-6\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = -\frac{216}{7}$ 		

QNo.		Solution	Marks	Comments
9	(c)	Applying $\mathbf{S} = \mathbf{u}t$ $\vec{AP} = (2\mathbf{i} + \mathbf{j})t$ $\mathbf{r} = (2\mathbf{i} + \mathbf{j})t - \mathbf{i} - \mathbf{j}$ $\mathbf{r} = (2t - 1)\mathbf{i} + (t - 1)\mathbf{j}$ Appliying $\mathbf{S} = \mathbf{u}t$ $\vec{AQ} = (\mathbf{i} - 2\mathbf{j})t$ $\mathbf{r} = (\mathbf{i} - 2\mathbf{j})t + 4\mathbf{i} + 4\mathbf{j}$ $\mathbf{r} = (4 + t)\mathbf{i} + (4 - 2t)\mathbf{j}$  $\therefore$ The distance between two paths at any time t is $\sqrt{(2t - 1 - 4 - t)^2 + (t - 1 - 4 + 2t)^2}$ $= \sqrt{10t^2 - 40t + 50} = \sqrt{10[(t-2)^2 + 1]}$ $\therefore\ d_{\min} = \sqrt{10}$ when $t = 2$ $\mathbf{V}_{PE} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{V}_{QE} = \mathbf{i} - 2\mathbf{j}$ $\mathbf{V}_{PQ} = \mathbf{V}_{PE} + \mathbf{V}_{EQ} = 2\mathbf{i} + \mathbf{j} + (-\mathbf{i} + 2\mathbf{j})$ $\mathbf{V}_{PQ} = \mathbf{i} + 3\mathbf{j}$		

Q No		Solution	Marks	Remarks
10	(a)	$\vec{OP} = \mathbf{r} = r\cos\theta\,\mathbf{i} + r\sin\theta\,\mathbf{j}$ The unit vector along the direction $\vec{OP}$ is $\frac{1}{r}\mathbf{r}$ $\therefore\ \boldsymbol{l} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$ $\vec{NM} = -r\sin\theta\tan\theta\,\mathbf{i} = -r\dfrac{\sin^2\theta}{\cos\theta}\mathbf{i}$ $\vec{MP} = r\sin\theta\,\mathbf{j}$ $\vec{NP} = \vec{NM} + \vec{MP}$ $\quad = -r\dfrac{\sin^2\theta}{\cos\theta}\mathbf{i} + r\sin\theta\,\mathbf{j}$ $\vec{MP} = r\tan\theta(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$ $\left\lvert \vec{MP} \right\rvert = r\tan\theta \quad \therefore\ \boldsymbol{m} = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$ $\dot{\boldsymbol{l}} = \dfrac{d\boldsymbol{l}}{dt} = \dfrac{d}{dt}(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$ $= -\sin\theta\dot{\theta}\,\mathbf{i} + \cos\theta\dot{\theta}\,\mathbf{j}$ $\dot{\boldsymbol{l}} = \dot{\theta}(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}) = \dot{\theta}\,\boldsymbol{m}$ $\dot{\boldsymbol{m}} = \dfrac{d\boldsymbol{m}}{dt} = \dfrac{d}{dt}(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$ $= -\cos\theta\dot{\theta}\,\mathbf{i} - \mathbf{sin}\theta\dot{\theta}\,\mathbf{j}$		

Q No		Solution	Marks	Remarks
		$\dot{\boldsymbol m} = -\dot\theta(\cos\theta\,\mathbf i + \sin\theta\,\mathbf j) = -\dot\theta\,\boldsymbol l$ $\mathbf r = r\boldsymbol l$ $\frac{d\mathbf r}{dt} = r\frac{d\boldsymbol l}{dt} + \frac{dr}{dt}\boldsymbol l$ $\dot{\mathbf r} = r\dot{\boldsymbol l} + \dot r\,\boldsymbol l$ $\therefore\ \dot{\mathbf r} = r\dot\theta\,\boldsymbol m + \dot r\,\boldsymbol l$ $\frac{d\dot{\mathbf r}}{dt} = \dot r\dot\theta\,\boldsymbol m + r\ddot\theta\,\boldsymbol m + r\dot\theta\,\dot{\boldsymbol m} + \ddot r\,\boldsymbol l + \dot r\,\dot{\boldsymbol l}$ $\ddot{\mathbf r} = \dot r\dot\theta\,\boldsymbol m + r\ddot\theta\,\boldsymbol m + r\dot\theta(-\dot\theta\,\boldsymbol l) + \ddot r\,\boldsymbol l + \dot r\dot\theta\,\boldsymbol m$ $= \left(\ddot r - r\dot\theta^2\right)\boldsymbol l + \left(r\ddot\theta + 2\dot r\dot\theta\right)\boldsymbol m$ $= \left(\ddot r - r\dot\theta^2\right)\boldsymbol l + \frac{1}{r}\left(r^2\ddot\theta + 2r\dot r\dot\theta\right)\boldsymbol m$ $= \left(\ddot r - r\dot\theta^2\right)\boldsymbol l + \frac{1}{r}\left(r^2\ddot\theta + 2r\dot r\dot\theta\right)\boldsymbol m$ $= \left(\ddot r - r\dot\theta^2\right)\boldsymbol l + \frac{1}{r}\left(\frac{d}{dt}\left(r^2\dot\theta\right)\right)\boldsymbol m$		
	(b)	Applying $\mathbf F = m\mathbf a$ For $M\downarrow$ $-T + Mg = M\frac{d^2}{dt^2}(2l - r) = -M\ddot r \cdots\cdots\cdots[1]$ For m along the string $-T = \left(m\ddot r - r\dot\theta^2\right)\cdots\cdots\cdots[2]$ For m along the tangent $0 = m\frac{1}{r}\frac{d^2(r^2\dot\theta)}{dt^2}\cdots[3]$ Integrating w.r.t. time $r^2\dot\theta = constant$ At $t = 0$ $\dot\theta = \frac{u}{a}$ $\therefore$ $constant = au^2$ $\therefore\ r^2\dot\theta = au^2$ $\dot\theta = \frac{au^2}{r^2}$ From [1] $\ddot r = \frac{T - Mg}{M}$ From [2] $-T = \left(\frac{T - Mg}{M} - \frac{ra^2u^2}{r^4}\right)$ $T = \frac{Mm}{M+m}\left(g + \frac{a^2u^2}{r^3}\right)$  		

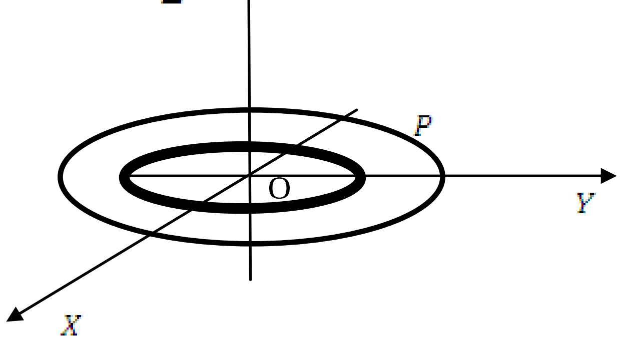
Q No		Solution	Marks	Remarks
11				



Consider a small part of the ring at P with mass δm

The moments of inertia of the ring about OZ

$= \sum \delta m a^2 = a^2 \sum \delta m = Ma^2$



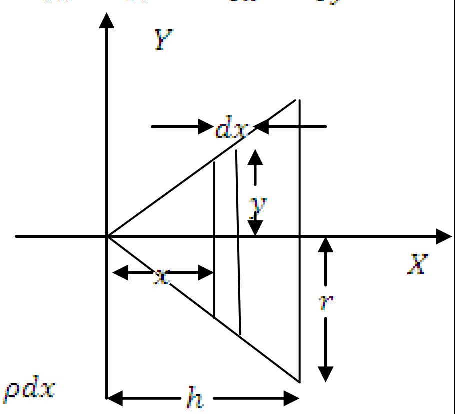
The area density of the plate $= \sigma = \frac{M}{\pi a^2}$

The moments of inertia about

$OZ = I_{OZ} = \int_0^a 2\pi x \sigma x^2\, dx = 2\pi\sigma \left[\frac{x^4}{4}\right]_0^a = \frac{1}{2} M a^2$

Using the perpendicular theorem $I_{OZ} = I_{OX} + I_{OY}$ and $I_{OX} = I_{Oy}$

$I_{OX} = \dfrac{1}{4} M a^2$



Now consider r the cone

$\rho =$ volume density

$\dfrac{y}{x} = \dfrac{r}{h}$

$y = \dfrac{r}{h} x$

$M = \dfrac{1}{3}\pi r^2 h \rho$

Mass of the plate $= \pi y^2 \rho dx = \pi \frac{r^2 x^2}{h^2} \rho dx$

The moments of inertia of the plate about its diameter

$= \frac{1}{4}\pi \frac{r^2 x^2}{h^2} \rho \frac{r^2 x^2}{h^2} dx$

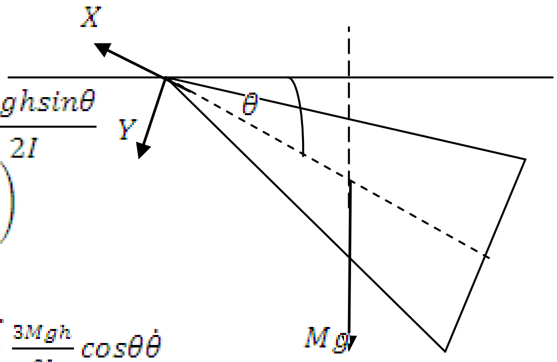
$= \dfrac{\pi \rho r^4 x^4}{4h^4} dx$

Using the parallel axis theorem the moments of inertia of the about OY

$= \dfrac{\pi r^2 \rho x^2}{h^2} x^2 dx + \dfrac{\pi \rho r^4 x^4}{4h^4} dx = \dfrac{\pi \rho r^2}{h^2}\left(x^4 + \dfrac{r^2 x^4}{4h^2}\right) dx$

The moments of inertia of the cone about $OY = \int_0^h \frac{\pi \rho r^2}{h^2}\left(x^2 + \frac{r^2 x^4}{4h^2}\right) dx$

$= \dfrac{\pi \rho r^2}{4h^2}(4h^2 + r^2)\left[\dfrac{x^5}{5}\right]_0^h = \dfrac{\pi r^2 \rho h}{20}(4h^2 + r^2) = \dfrac{3M}{20}(4h^2 + r^2)$

Q No	Solution	Marks	Remarks
	Applying the energy conservation law $\frac{1}{2}I\dot{\theta}^2 - mg\frac{3h}{4}\sin\theta = 0 \ldots\ldots [1]$ Applying $\boldsymbol{F} = Ma$ $X - Mg\sin\theta = M\frac{3h}{4}\dot{\theta}^2$ $X = Mg\sin\theta + M\frac{3h}{4}\frac{3Mgh\sin\theta}{2I}$ $X = Mg\sin\theta\left(1 + \frac{9Mh^2}{8I}\right)$ $Y + Mg\cos\theta = M\frac{3h}{4}\ddot{\theta}$ Differentiating [1] $2\theta\ddot{\theta} = \frac{3Mgh}{2I}\cos\theta\dot{\theta}$ $\ddot{\theta} = \frac{3Mgh}{4I}\cos\theta$ $Y = -Mg\cos\theta + \frac{3Mh}{4}\frac{3Mgh}{4I}\cos\theta$ $= \left(\frac{9h^2}{16I} - 1\right)Mgh\cos\theta$ When axis is vertical $\theta = \frac{\pi}{2}$ $Y = 0$ $X = Mg\left(1 + \frac{9Mh^2}{8I}\right)$ $X = Mg\left(1 + \frac{9Mh^2}{8\frac{3M}{20}(r^2 + 4h^2)}\right)$ $X = Mg\left(1 + \frac{15h^2}{2(r^2 + 4h^2)}\right)$ 		

Q No	Solution	Marks	Remarks
12(a)	$\mathbf{A} = 2t\mathbf{i} - (t-1)\mathbf{j} + e^t\mathbf{k}\ ,\ \mathbf{B} = (t-1)\mathbf{i} + 2t\mathbf{j} - e^t\mathbf{k}$ $\mathbf{A}\cdot\mathbf{B} = 2t(t-1) - 2t(t-1) + +e^t(-e^t) = -e^{2t}$ $\frac{d}{dt}(\mathbf{A}\cdot\mathbf{B}) = -2e^{2t}$ $\frac{d}{dt}(\mathbf{A}\times\mathbf{B}) = \frac{d}{dt}\mathbf{A}\times\mathbf{B} + \mathbf{A}\times\frac{d}{dt}\mathbf{B}$ $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & e^t \\ t-1 & 2t & -e^t \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2t & -(t-1) & e^t \\ 1 & 2 & e^t \end{vmatrix} = -(t+2)e^t\mathbf{i} + (2+3t)e^t\mathbf{j} + 8t\mathbf{k}$ $\frac{d}{dt}(\mathbf{A}+\mathbf{B}) = \frac{d}{dt}\mathbf{A} + \frac{d}{dt}\mathbf{B} = \frac{d}{dt}(2t\mathbf{i} - (t-1)\mathbf{j} + e^t\mathbf{k} + \frac{d}{dt}((t-1)\mathbf{i} + 2t\mathbf{j} - e^t\mathbf{k}$ $= -2\mathbf{i} - \mathbf{j} + e^t\mathbf{k} + \mathbf{i} + 2\mathbf{j} - e^t\mathbf{k} = 3\mathbf{i} + \mathbf{j}$		

Q No	Solution	Marks	Remarks
12(b)	$\frac{d}{dt}(\mathbf{r}\times(\mathbf{v}\times\mathbf{r}))=\frac{d}{dt}[(\mathbf{r}\cdot\mathbf{r})\mathbf{v}-(\mathbf{v}\cdot\mathbf{r})\mathbf{r}]$ $\frac{d}{dt}(\mathbf{r}\times(\mathbf{v}\times\mathbf{r}))=\frac{d}{dt}[\mathbf{r}\cdot\mathbf{r}]\mathbf{v}+(\mathbf{r}\cdot\mathbf{r})\frac{d}{dt}\mathbf{v}-\frac{d}{dt}[\mathbf{v}\cdot\mathbf{r}]\mathbf{r}-[\mathbf{v}\cdot\mathbf{r}]\frac{d}{dt}\mathbf{r}$ $=[\mathbf{r}\cdot\frac{d}{dt}\mathbf{r}+\mathbf{r}\cdot\frac{d}{dt}\mathbf{r}]\mathbf{v}+r^2\mathbf{a}$ $-[\mathbf{v}\cdot\frac{d}{dt}\mathbf{r}+\frac{d}{dt}\mathbf{v}\cdot\mathbf{r}]\mathbf{r}-[\mathbf{v}\cdot\mathbf{r}]\mathbf{v}$ $=2(\mathbf{v}\cdot\mathbf{r})\mathbf{v}+r^2\mathbf{a}-[\mathbf{v}\cdot\mathbf{v}+\mathbf{a}\cdot\mathbf{r}]\mathbf{r}-(\mathbf{v}\cdot\mathbf{r})\mathbf{v}$ $=r^2\mathbf{a}+(\mathbf{v}\cdot\mathbf{r})\mathbf{v}-[v^2+\mathbf{a}\cdot\mathbf{r}]\mathbf{r}$		
12(c)	(i) $\mathbf{f}(t)=\frac{\cos t}{\sqrt{1+e^{2t}}}\mathbf{i}+\frac{\sin t}{\sqrt{1+e^{2t}}}\mathbf{j}+\frac{e^t}{\sqrt{1+e^{2t}}}\mathbf{k}$ $\lvert\mathbf{f}(t)\rvert=\sqrt{\left(\frac{\cos t}{\sqrt{1+e^{2t}}}\right)^2+\left(\frac{\sin t}{\sqrt{1+e^{2t}}}\right)^{\mathbf{2}}+\left(\frac{e^t}{\sqrt{1+e^{2t}}}\right)^{\mathbf{2}}}$ $=\sqrt{\frac{\cos^2 t}{1+e^{2t}}+\frac{\sin^2 t}{1+e^{2t}}+\frac{e^{2t}}{1+e^{2t}}}=1$ $\lvert\mathbf{f}(t)\rvert=1$. $\lvert\mathbf{f}(t)\rvert^2=1$ $\mathbf{f}(t)\cdot\mathbf{f}(t)=1$ $\frac{d}{dt}[\mathbf{f}(t)\cdot\mathbf{f}(t)]=0$ $\mathbf{f}(t)\cdot\frac{d}{dt}\mathbf{f}(t)+\mathbf{f}(t)\cdot\frac{d}{dt}\mathbf{f}(t)=0$ $2\mathbf{f}(t)\cdot\frac{d}{dt}\mathbf{f}(t)=0$ $\mathbf{f}(t)\cdot\frac{d}{dt}\mathbf{f}(t)=0$ $\therefore$ $\mathbf{f}(t)$ is perpendicular to $\frac{d}{dt}(\mathbf{f}(t))$.		